Wood Beam Analysis and Design

- ASD approach
- NDS criteria
- Wood Beam Analysis
- Wood Beam Design

Allowable Stress Design

Allowable Stress ≥ Actual Stress

\[ F_b \text{ from the NDS Supplement} \]
Allowable Stress Design by NDS

**Flexure**

\[ F_b' \geq f_b \]

**Allowable Flexure Stress** \( F_b' \)

- \( F_b \) from NDS Supplement tables determined by species and grade
- \( F_b' = F_b \) (usage factors)
  - usage factors for flexure:
    - \( C_D \) Load Duration Factor
    - \( C_M \) Moisture Factor
    - \( C_T \) Temperature Factor
    - \( C_B \) Beam Stability Factor
    - \( C_S \) Size Factor
    - \( C_F \) Flat Use
    - \( C_I \) Incising Factor
    - \( C_R \) Repetitive Member Factor

\[ f_b = \frac{Mc}{I} = \frac{M}{S} \]

- \( S = \frac{I}{c} = bd^2/6 \)

**Actual Flexure Stress** \( f_b \)

Allowable Stress Design by NDS

**Shear**

\[ F_v' \geq f_v \]

**Allowable Shear Stress** \( F_v' \)

- \( F_v \) from tables determined by species and grade
- \( F_v' = F_v \) (usage factors)
  - usage factors for shear:
    - \( C_D \) Load Duration Factor
    - \( C_M \) Moisture Factor
    - \( C_T \) Temperature Factor
    - \( C_I \) Incising Factor

\[ f_v = \frac{VQ}{Ib} = 1.5 \frac{V}{A} \]

- Can use \( V \) at \( d \) from support as maximum
Allowable Stress Design by NDS Compression

**F′ _c** = **F_c** (usage factors)

**usage factors for flexure:**
- **C_D** Load Duration Factor
- **C_M** Moisture Factor
- **C_T** Temperature Factor
- **C_i** Size Factor
- **C_i** Incising Factor
- **C_p** Column Stability Factor

**Actual Compression Stress f_c**

\[ f_c = \frac{P}{A} \]

**Allowable Compression Stress F′ _c**

**F_c** from NDS Supplement tables determined by species and grade

Adjustment Factors

**Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber**

<table>
<thead>
<tr>
<th>Adjustment Factor</th>
<th>ASD only</th>
<th>ASD and LRFD</th>
<th>LRFD only</th>
</tr>
</thead>
<tbody>
<tr>
<td>F_b = F_b</td>
<td>x</td>
<td>C_D C_M C_t C_L C_p C_i C_t</td>
<td>K_F φ_b λ</td>
</tr>
<tr>
<td>F_i = F_i</td>
<td>x</td>
<td>C_D C_M C_t C_p C_i C_i C_i</td>
<td>K_F φ_i λ</td>
</tr>
<tr>
<td>F_p = F_p</td>
<td>x</td>
<td>C_D C_M C_t C_p C_i C_i C_i C_i</td>
<td>K_F φ_i λ</td>
</tr>
<tr>
<td>F_c = F_c</td>
<td>x</td>
<td>C_D C_M C_t C_p C_i C_i C_i C_i</td>
<td>K_F φ_c λ</td>
</tr>
<tr>
<td>F_p = F_p</td>
<td>x</td>
<td>C_D C_M C_t C_p C_i C_i C_i C_i</td>
<td>K_F φ_i λ</td>
</tr>
<tr>
<td>E = E</td>
<td>x</td>
<td>C_D C_M C_t C_p C_i C_i C_i C_i</td>
<td>- - - -</td>
</tr>
<tr>
<td>E_min = E_min</td>
<td>x</td>
<td>C_D C_M C_t C_p C_i C_i C_i C_i</td>
<td>K_F φ_i -</td>
</tr>
</tbody>
</table>
Adjustment Factors

Allowable Flexure Stress $F_{b'}$

$F_{b'} = F_b \left( C_D C_M C_t C_L C_F C_{fu} C_i C_r \right)$

Usage factors for flexure:
- $C_D$: Load Duration Factor
- $C_t$: Temperature Factor

Table 2.3.2 Frequently Used Load Duration Factors, $C_D$

<table>
<thead>
<tr>
<th>Load Duration</th>
<th>$C_D$</th>
<th>Typical Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent</td>
<td>0.9</td>
<td>Dead Load</td>
</tr>
<tr>
<td>Ten years</td>
<td>1.0</td>
<td>Occupancy Live Load</td>
</tr>
<tr>
<td>Two months</td>
<td>1.15</td>
<td>Snow Load</td>
</tr>
<tr>
<td>Seven days</td>
<td>1.25</td>
<td>Construction Load</td>
</tr>
<tr>
<td>Ten minutes</td>
<td>1.6</td>
<td>Wind/Earthquake Load</td>
</tr>
<tr>
<td>Impact^2</td>
<td>2.0</td>
<td>Impact Load</td>
</tr>
</tbody>
</table>

(1) Actual stress due to (DL) ≤ (0.9) (Design value)
(2) Actual stress due to (DL+LL) ≤ (1.0) (Design value)
(3) Actual stress due to (DL+WL) ≤ (1.6) (Design value)
(4) Actual stress due to (DL+LL+SL) ≤ (1.15) (Design value)
(5) Actual stress due to (DL+LL+WL) ≤ (1.6) (Design value)
(6) Actual stress due to (DL+SL+WL) ≤ (1.6) (Design value)
(7) Actual stress due to (DL+LL+SL+WL) ≤ (1.6) (Design value)

2018 NDS

Table 2.3.3 Temperature Factor, $C_t$

<table>
<thead>
<tr>
<th>Reference Design Values</th>
<th>In-Service Moisture Condition</th>
<th>$C_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{t, E_{max}}$ Wet or Dry</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>$F_{t, E_{min}}$ Dry</td>
<td>1.0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

* We will ignore conditions for users' loads, moisture load, fire load, wind loads, applied composite loads, wood structural panels and cross-laminated timber are specified as A1.4, A1.4.1, A1.4.2, A1.4.3, and 11.2.2, respectively.

Adjustment Factors

Allowable Flexure Stress $F_{b'}$

$F_{b'} = F_b \left( C_D C_M C_t C_L C_F C_{fu} C_i C_r \right)$

Usage factors for flexure:
- $C_M$: Moisture Factor
- $C_F$: Size Factor

Table 2.3.4 Wet Service Factor, $C_M$

<table>
<thead>
<tr>
<th>Wet Service Factors, $C_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:</td>
</tr>
<tr>
<td>$F_b$</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>0.85*</td>
</tr>
</tbody>
</table>

* when (Fb)(Ct) ≤ 1,150 psi, Cm = 1.0
** when (Fb)(Ct) ≤ 750 psi, Cm = 1.0

Table 2.3.5 Size Factors, $C_s$

<table>
<thead>
<tr>
<th>Grades</th>
<th>Width (depth)</th>
<th>Thickness (breadth)</th>
<th>$F_s$</th>
<th>$F_t$</th>
<th>$F_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select</td>
<td>2&quot;, 3&quot;, &amp; 4&quot;</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.15</td>
</tr>
<tr>
<td>Structural,</td>
<td>6&quot;</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>1.1</td>
</tr>
<tr>
<td>No.1 &amp; Btr,</td>
<td>8&quot;</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.05</td>
</tr>
<tr>
<td>No.1, No.2,</td>
<td>10&quot;</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>No.3</td>
<td>12&quot;</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>14&quot; &amp; wider</td>
<td>0.9</td>
<td>1.0</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Stud</td>
<td>5&quot; &amp; 6&quot;</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>8&quot; &amp; wider</td>
<td>Use No.3 Grade tabulated design values and size factors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Construction,</td>
<td>2&quot;, 3&quot;, &amp; 4&quot;</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Utility</td>
<td>4&quot;</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>2&quot; &amp; 3&quot;</td>
<td>0.4</td>
<td>—</td>
<td>0.4</td>
<td>0.4</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Adjustment Factors

Allowable Flexure Stress $F_{b}'$

$F_{b}' = F_{b} (C_D C_M C_t C_L C_r C_{fu} C_i C_r)$

Usage factors for flexure:
- $C_{fu}$ Flat Use
- $C_r$ Repetitive Member Factor

Flat Use Factor, $C_{fu}$

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value, $F_{bu}$, shall also be permitted to be multiplied by the following flat use factors:

<table>
<thead>
<tr>
<th>Width (depth)</th>
<th>Thickness (breadth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2&quot; &amp; 3&quot;</td>
<td>1.0</td>
</tr>
<tr>
<td>4&quot;</td>
<td>1.1</td>
</tr>
<tr>
<td>5&quot;</td>
<td>1.1</td>
</tr>
<tr>
<td>6&quot;</td>
<td>1.15</td>
</tr>
<tr>
<td>8&quot;</td>
<td>1.15</td>
</tr>
<tr>
<td>10&quot; &amp; wider</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Repetitive Member Factor, $C_r$

Bending design values, $F_{br}$, for dimension lumber 2" to 4" thick shall be multiplied by the repetitive member factor, $C_r = 1.15$, when such members are used as joists, truss chords, rafters, studs, planks, decking, or similar members which are in contact or spaced not more than 24" on center, are not less than 3 in number and are joined by floor, roof, or other load distributing elements adequate to support the design load.

Adjustment Factors

Allowable Flexure Stress $F_{b}'$

$F_{b}' = F_{b} (C_D C_M C_t C_L C_r C_{fu} C_i C_r)$

Usage factors for flexure:
- $C_i$ Incising Factor

Table 4.3.8 Incising Factors, $C_i$

<table>
<thead>
<tr>
<th>Design Value</th>
<th>$C_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$, $E_{min}$</td>
<td>0.95</td>
</tr>
<tr>
<td>$F_{bu}$, $F_{b}$, $F_{v}$</td>
<td>0.80</td>
</tr>
<tr>
<td>$F_{cu}$</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Adjustment Factors

Allowable Flexure Stress $F_b'$

$F_b' = F_b \left( C_D C_M C_I C_L C_F C_{tu} C_i C_r \right)$

Usage factors for flexure:

$C_L$ Beam Stability Factor

### 3.3.3 Beam Stability Factor, $C_L$

3.3.3.1 When the depth of a bending member does not exceed its breadth, $d \leq b$, no lateral support is required and $C_L = 1.0$.

3.3.3.2 When rectangular sawn lumber bending members are laterally supported in accordance with 4.4.1, $C_L = 1.0$.

3.3.3.3 When the compression edge of a bending member is supported throughout its length to prevent lateral displacement, and the ends at points of bearing have lateral support to prevent rotation, $C_L = 1.0$.

2012 NDS

---

### 4.4.1 Stability of Bending Members

2x4 (a) $d/b \leq 2$; no lateral support shall be required.

2x6-8 (b) $2 < d/b \leq 4$; the ends shall be held in position, as by full depth solid blocking, bridging, hang- ers, nailing, or bolting to other framing mem- bers, or other acceptable means.

2x10 (c) $4 < d/b \leq 5$; the compression edge of the member shall be held in line for its entire length to prevent lateral displacement, as by adequate sheathing or subflooring, and ends at point of bearing shall be held in position to prevent rotation and/or lateral displacement.

2x12 (d) $5 < d/b \leq 6$; bridging, full depth solid blocking or diagonal cross bracing shall be installed at intervals not exceeding 8 feet, the compression edge of the member shall be held in line as by adequate sheathing or subflooring, and the ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

2x14 (e) $6 < d/b \leq 7$; both edges of the member shall be held in line for their entire length and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

---

$C_L$

$C_L = 1.0$

for depth/width ratio in

4.4.1 $C_L = 1.0$

Otherwise

$C_L < 1.0$

calculate factor using section 3.3.3

<table>
<thead>
<tr>
<th>Beam Depth/Width Ratio</th>
<th>Type of Lateral Bracing Required</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 to 1</td>
<td>None</td>
<td><img src="image" alt="2x4 Beam" /></td>
</tr>
<tr>
<td>3 to 1</td>
<td>The ends of the beam should be held in position</td>
<td><img src="image" alt="3x6 Beam" /></td>
</tr>
<tr>
<td>2x6</td>
<td></td>
<td><img src="image" alt="2x8 Beam" /></td>
</tr>
<tr>
<td>2x8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 to 1</td>
<td>Hold compression edge in line (continuously)</td>
<td><img src="image" alt="5x10 Beam" /></td>
</tr>
<tr>
<td>2x10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 to 1</td>
<td>Diagonal bridging should be used</td>
<td><img src="image" alt="6x12 Beam" /></td>
</tr>
<tr>
<td>2x12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 to 1</td>
<td>Both edges of the beam should be held in line</td>
<td><img src="image" alt="7x14 Beam" /></td>
</tr>
<tr>
<td>2x14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**CL Beam Stability Factor**

In the case bracing provisions of 4.4.1 cannot be met, \( C_L \) is calculated using equation 3.3.6

The maximum allowable slenderness, \( R_b \) is 50

<table>
<thead>
<tr>
<th>Table 3.3.3 Effective Length, ( L_e ) for Bending Members</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Uniformly distributed load</td>
</tr>
<tr>
<td>Concentrated load at unsupported end</td>
</tr>
</tbody>
</table>

Single Span Beam**1**

| Parameter | \( L_e < 7 \) | \( L_e \geq 7 \) |
| Uniformly distributed load | \( L_e = 2.06 L_e \) | \( L_e = 1.63 L_e + 3d \) |
| Concentrated load at center with no intermediate lateral support | \( L_e = 1.80 L_e \) | \( L_e = 1.37 L_e + 3d \) |

3.3.3.6 The slenderness ratio, \( R_b \), for bending members shall be calculated as follows:

\[
R_b = \frac{L_e d}{\sqrt{B_i}} \tag{3.3-5}
\]

3.3.3.7 The slenderness ratio for bending members, \( R_b \), shall not exceed 50.

3.3.3.8 The beam stability factor shall be calculated as follows:

\[
C_L = \frac{1 + \left( F_{v_i}/F_v \right)}{1.9} - \left[ 1 + \left( F_{v_i}/F_v \right) \right]^{-0.95} \tag{3.3-6}
\]

where:

- \( F_{v_i} \) is the reference bending design value multiplied by all applicable adjustment factors except \( C_{v_o} \)
- \( C_v \) (when \( C_v \leq 1.0 \)), and \( G \) (see 2.3), psi

\[
F_{v_i} = \frac{1.20 F_{v_{	ext{design}}}}{R_e^2}
\]

---

**Adjustment Factors for Shear**

**Allowable Flexure Stress \( F_v' \)**

\( F_v \) from tables determined by species and grade

\( F_v' = F_v \) (usage factors)

Usage factors for shear:

- \( C_D \): Load Duration Factor
- \( C_M \): Moisture Factor
- \( C_t \): Temperature Factor
- \( C_i \): Incising Factor

**Shear at Supports**

Modified shear \( V' \) used to compute reduced shear \( f' \).
Analysis Procedure

Given: loading, member size, material and span.

Req’d: Safe or Unsafe

1. Find Max Shear & Moment
   - Simple case – equations
   - Complex case - diagrams

2. Determine actual stresses
   - \( f_b = \frac{M}{S} \)
   - \( f_v = 1.5 \frac{V}{A} \)

3. Determine allowable stresses
   - \( F_b \) and \( F_v \) (from NDS)
   - \( F_b’ = F_b \) (usage factors)
   - \( F_v’ = F_v \) (usage factors)

4. Check that actual \( \leq \) allowable
   - \( f_b \leq F_b’ \)
   - \( f_v \leq F_v’ \)

5. Check deflection

6. Check bearing \( (F_{cul} \geq \frac{Reaction}{A_{bearing}}) \) from NDS 2012

Analysis Example

Example

Given: loading, member size, material and span.

Req’d: Safe or Unsafe?
Analysis Example

1. Find Max Shear & Moment
   - Simple cases – equations
   - Complex cases - diagrams

By equations:

\[ V = \frac{P}{2} = \frac{145}{2} = 72.5 \text{ kips} \]

\[ M_x = \frac{PL}{4} = \frac{145(69)}{4} \]

\[ M_x = 217.5 \text{ kips*ft} \]

By Diagrams:

2. Determine actual stresses
   - \( f_b = \frac{M_x}{S_x} \)
   - \( f_v = 1.5 \frac{V}{A} \)

\[ f_b = \frac{217.5}{3,063} = 0.07 \text{ ksi} \]

\[ f_v = \frac{3}{2} \times \frac{72.5}{5.25} = 20.71 \text{ psi} \]
Species and Grade

S-P-F  No.2

$F_b = 875$ psi

$F_v = 135$ psi

### Analysis Example

3. Determine allowable stresses
   - $F_b = 875$ psi
   - $F_v = 135$ psi

Determine factors:

| CD = ? | CM = 1 | Ct = 1 | CL = 1 | CF = ? | Cfu = 1 | Ci = 1 | Cr = 1 |

### Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

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<thead>
<tr>
<th>ASD only</th>
<th>ASD and LRFD</th>
<th>LRFD only</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_b$</td>
<td>$F_v$</td>
<td>$K_F$</td>
</tr>
<tr>
<td>$C_D$</td>
<td>$C_M$</td>
<td>$C_I$</td>
</tr>
<tr>
<td>$C_L$</td>
<td>$C_F$</td>
<td>$C_{fu}$</td>
</tr>
<tr>
<td>$C_I$</td>
<td>$C_T$</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>$K_F$</td>
<td>$\phi_v$</td>
<td>$\lambda$</td>
</tr>
</tbody>
</table>

---

University of Michigan, TCAUP

Structures II
Analysis Example

**C\(_D\)** Load duration factor

Use 1.6 (10 minutes)

**C\(_F\)** Size factor

2 x 4

use 1.5

---

### Frequently Used Load Duration Factors, C\(_D\)^4

<table>
<thead>
<tr>
<th>Load Duration</th>
<th>C(_D)</th>
<th>Typical Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent</td>
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<td>Snow Load</td>
</tr>
<tr>
<td>Seven days</td>
<td>1.25</td>
<td>Construction Load</td>
</tr>
<tr>
<td>Ten minutes</td>
<td>1.6</td>
<td>Wind/Earthquake Load</td>
</tr>
<tr>
<td>Impact(^1)</td>
<td>2.0</td>
<td>Impact Load</td>
</tr>
</tbody>
</table>

---

3. **Determine allowable stresses**

- \( F'\_b = F\_b (C\(_D\))(C\(_F\)) \)
- \( F'\_b = 875 (1.6)(1.5) = 2100 \text{ psi} \)

- \( F'\_v = F\_v (C\(_D\)) \)
- \( F'\_v = 135 (1.6) = 216 \text{ psi} \)

4. **Check that actual < allowable**

- \( f\_b < F'\_b \)
- \( f\_v < F'\_v \)

5. **Check deflection**

6. **Check bearing** (\( F\_c\_\perp \geq R/A\_b \))
Analysis Procedure

Given: member size, material and span.
Req’d: Max. Safe Load (capacity)

1. Assume \( f = F' \)
   - Maximum actual = allowable stress

2. Solve stress equations for force
   - \( M = F_b S \)
   - \( V = 0.66 F_v A \)

3. Use maximum moment to find loads
   - Back calculate a load from moment
   - Assumes moment controls

4. Check Shear
   - Use load found in step 3 to check shear stress.
   - If it fails (\( f_v > F'_v \)), then find load based on shear.

5. Check deflection
6. Check bearing

Analysis Example

Given: member size, material and span.
Req’d: Max. Safe Load (capacity)

1. Assume \( f = F' \)
   - Maximum actual = allowable stress

2. Solve stress equation for moment
   - \( M = F'_b S \) (i.e. moment capacity)
Analysis Example (cont.)

3. Use maximum forces to find loads
   • Back calculate a maximum load from moment capacity

4. Check shear
   • Check shear for load capacity from step 3.
   • Use P from moment to find Vmax
   • Check that $f_v < F_v$

4. Check deflection (serviceability)
5. Check bearing (serviceability)
## Analysis Example

Find Specific Gravity for Hem-Fir

- (from NDS)

### Table 4A

<table>
<thead>
<tr>
<th>Species and commercial grade</th>
<th>Size classification</th>
<th>Design values in pounds per square inch (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bending</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tenion parallel to grain F_1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shear perpendicular to grain F_2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Compression parallel to grain F_3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Modulus of Elasticity E</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Specific Gravity G</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Grading Rules Agency</td>
</tr>
<tr>
<td>Hem-Fir</td>
<td>Select Structural</td>
<td>1,600</td>
</tr>
<tr>
<td></td>
<td>No. 1 (8&quot; x 4&quot;)</td>
<td>1,100</td>
</tr>
<tr>
<td></td>
<td>No. 2 (8&quot; x 6&quot;)</td>
<td>975</td>
</tr>
<tr>
<td></td>
<td>No. 3 (8&quot; x 8&quot;)</td>
<td>850</td>
</tr>
<tr>
<td></td>
<td>Select Structural</td>
<td>797</td>
</tr>
<tr>
<td></td>
<td>No. 1 (6&quot; x 4&quot;)</td>
<td>750</td>
</tr>
<tr>
<td></td>
<td>No. 2 (6&quot; x 6&quot;)</td>
<td>725</td>
</tr>
<tr>
<td></td>
<td>No. 3 (6&quot; x 8&quot;)</td>
<td>690</td>
</tr>
</tbody>
</table>

### Section Properties:

- **4 x 12 (3.5" x 11.25")**

  - **Area = 39.38 in<sup>2</sup>**
  - **Sx = 73.83 in<sup>3</sup>**

---

**X-X AXIS**

- Nominal Size
- Standard Dressed Size (S4S)
- Area of Section A in<sup>2</sup>
- Section Modulus I<sub>x</sub> in<sup>4</sup>
- Moment of Inertia I<sub>x</sub> in<sup>4</sup>

**Y-Y AXIS**

- Nominal Size
- Standard Dressed Size (S4S)
- Area of Section A in<sup>2</sup>
- Section Modulus I<sub>y</sub> in<sup>4</sup>
- Moment of Inertia I<sub>y</sub> in<sup>4</sup>

---

**Design Values for Wood Construction – NDS Suplement**

---

**Use with Table 4A Adjustment Factors**

<table>
<thead>
<tr>
<th>Species and commercial grade</th>
<th>Size classification</th>
<th>Design values in pounds per square inch (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bending</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tenion parallel to grain F_1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shear perpendicular to grain F_2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Compression parallel to grain F_3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Modulus of Elasticity E</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Specific Gravity G</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Grading Rules Agency</td>
</tr>
</tbody>
</table>

---

**Analysis Example**

**Section Properties:**

- **4 x 12 (3.5" x 11.25")**

  - **Area = 39.38 in<sup>2</sup>**
  - **Sx = 73.83 in<sup>3</sup>**
Analysis Example

Determine Loading

• Find Tributary area, \( A_T \)
  \[ 6' \times 8' = 48 \text{ SF} \]

• Determine member selfweight \( (w) \)

\[
D_L = 7 \text{ PSF} \\
L_L = 35 \text{ PSF} \\
\text{Total} = 42 \text{ PSF} \\
p = A_T \times \text{PSF} \\
= 48 \times 42 = 2016 \text{ lb} \\
\]

Selfweight of member:

Density at 0 m.c. = 62.4 x G (dry)
\[ 62.4 \times 0.43 = 26.8 \text{ PCF} \]

To include m.c. use NDS formula.

\[
density = 62.4 \left[ \frac{G}{1 + G(0.009)(\text{m.c.})} \right] \frac{1 + \text{m.c.}}{100} \\
\]

where:

\[ G = \text{specific gravity of wood} \]
\[ \text{m.c.} = \text{moisture content of wood, \%} \]

\[
D = 62.4 \left[ \frac{0.43}{1 + 0.43(0.009)(15)} \right] \left[ 1 + \frac{15}{100} \right] \\
25.35 \times 1.15 = 29.16 \text{ PCF} \\
w = \text{PLF} = \frac{D \times \text{AREA}}{144} = \frac{29.16 \times 39.38}{144} \\
w = 7.975 \text{ PLF} \]
Analysis Example

Determine Beam Forces

by superposition equations or by diagrams

\[ R = \frac{w_1 l^2}{2} + \frac{p}{2} \]
\[ = \frac{7.97 \times l^2}{2} + \frac{2016}{2} \]
\[ = \frac{416.85}{2} + 1008 = 1055.8 \text{ kN} \]

\[ V_{max} = R \]
\[ M_{max} = \frac{w_1 l^2}{2} + \frac{p l}{4} \]
\[ = \frac{7.97 \times l^2}{2} + \frac{2016 l}{4} \]
\[ = 1415.5 + 6048 = 6915.5 \text{ kN.m} \]

Analysis Example

Determine actual stresses

\[ f_b = \frac{M}{S} \]
\[ f_v = 1.5 \frac{V}{A} \]

<table>
<thead>
<tr>
<th>Nominal Size</th>
<th>Standard Gaged Size (SGS)</th>
<th>Area of Section</th>
<th>Section Modulus</th>
<th>Section Modulus of Inertia</th>
<th>Moment of Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-AXIS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y-AXIS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dimension Lumber (see NDS 4.1.3.2 and Decoting (see NDS 4.1.3.5))

\[ f_b = \frac{M}{S} = \frac{6915.5 \times l}{93.83} = 100 M \text{ psi} \]
\[ f_v = \frac{3 \times V}{A} = 1.5 \frac{10558}{39.88} = 40.22 \text{ psi} \]
Analysis Example

Determine allowable stresses

- $F_b = 1400$ psi
- $F_v = 150$ psi

Determine factors:

$$CD = \quad CM = \quad Ct = \quad CL = \quad CF = \quad Cfu = \quad Cl = \quad Cr =$$

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

<table>
<thead>
<tr>
<th>ASD only</th>
<th>ASD and LRFD</th>
<th>LRFD only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load Duration Factor</td>
<td>Wet Service Factor</td>
<td>Temperature Factor</td>
</tr>
<tr>
<td>$F_b^\prime = F_b$</td>
<td>$F_v^\prime = F_v$</td>
<td>$x$</td>
</tr>
</tbody>
</table>

University of Michigan, TCAUP                                                                 Structures II                                                                 Slide 34 of 50
Analysis Example

Determine allowable stresses
M.C. = 15% size: 4x12

<table>
<thead>
<tr>
<th>Nominal Size</th>
<th>Standard Dressed Size (54S)</th>
<th>Area of Section</th>
<th>X-X Axis</th>
<th>Y-Y Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>b x d</td>
<td>b x d</td>
<td>A (in²)</td>
<td>Iₓ (in⁴)</td>
<td>Iᵧ (in⁴)</td>
</tr>
<tr>
<td>1 x 3</td>
<td>3/4 x 3-1/2</td>
<td>1.875</td>
<td>0.617</td>
<td>0.234</td>
</tr>
<tr>
<td>1 x 4</td>
<td>3/4 x 4-1/2</td>
<td>2.625</td>
<td>1.531</td>
<td>0.328</td>
</tr>
<tr>
<td>1 x 5</td>
<td>3/4 x 5-1/2</td>
<td>4.125</td>
<td>3.781</td>
<td>0.516</td>
</tr>
<tr>
<td>1 x 6</td>
<td>3/4 x 6-1/4</td>
<td>5.438</td>
<td>6.570</td>
<td>0.680</td>
</tr>
<tr>
<td>1 x 7</td>
<td>3/4 x 7-1/4</td>
<td>6.938</td>
<td>9.707</td>
<td>0.857</td>
</tr>
<tr>
<td>1 x 8</td>
<td>3/4 x 8-1/4</td>
<td>8.436</td>
<td>12.62</td>
<td>1.055</td>
</tr>
</tbody>
</table>

**Adjustment Factors**

Allowable Flexure Stress $F_b'$

$F_b' = F_b \left( C_D \cdot C_M \cdot C_1 \cdot C_F \cdot C_{fu} \cdot C_i \cdot C_r \right)$

$b/d = 3.5 / 11.25 = 0.311$ (case b)

Assuming ends are braced, $C_L = 1.0$

3.3.3 Beam Stability Factor, $C_L$

3.3.3.1 When the depth of a bending member does not exceed its breadth, $d \leq b$, no lateral support is required and $C_L = 1.0$.

3.3.3.2 When rectangular sawn lumber bending members are laterally supported in accordance with 4.4.1, $C_L = 1.0$.

3.3.3.3 When the compression edge of a bending member is supported throughout its length to prevent lateral displacement, and the ends at points of bearing have lateral support to prevent rotation, $C_L = 1.0$.

2012 NDS

4.4.1 Stability of Bending Members

2x4 (a) $d/b \leq 2$; no lateral support shall be required.

2x6-8 (b) $2 < d/b \leq 4$; the ends shall be held in position; as by full depth solid blocking, bridging, hangars, nailing, or bolting to other framing members, or other acceptable means.

2x10 (c) $4 < d/b \leq 5$; the compression edge of the member shall be held in line for its entire length to prevent lateral displacement, as by adequate sheathing or subflooring, and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

2x12 (d) $5 < d/b \leq 6$; bridging, full depth solid blocking or diagonal cross bracing shall be installed at intervals not exceeding 8 feet, the compression edge of the member shall be held in line as by adequate sheathing or subflooring, and the ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

2x14 (e) $6 < d/b \leq 7$; both edges of the member shall be held in line for their entire length and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.
Analysis Example

3. Determine allowable stresses
   - $F_{b}' = F_b$ (usage factors)

\[
\begin{align*}
F_b & = F_b \\
C_D & = 1.0 \quad \text{(live load)} \\
C_{1,0} & = 1.0 \quad \text{15\%} < 19\% \quad \text{(NDS Sec. P.38)} \\
C_I & = 1.0 \quad \text{temp.} < 100^\circ \\
C_L & = 1.0 \quad \text{ENACED for } A14.1 \\
C_F & = 1.0 \quad \text{FOR } A \times 12 \quad \text{(NDS Sec. P.32)} \\
C_{f_u} & = 1.0 \quad \text{NOT} \quad \text{(NDS Sec. P.32)} \\
C_i & = 1.0 \quad \text{NOT} \quad \text{(NDS Sec. P.32)} \\
C_r & = 1.0 \quad \text{NOT} \quad \text{(NDS Sec. P.32)}
\end{align*}
\]

$F_b' = 1400 (1.1) = 1540 \text{ psi}$

Analysis Example

3. Determine allowable stresses
   - $F_v' = F_v$ (usage factors)

\[
\begin{align*}
F_v & = F_v \\
C_D & = 1.0 \\
C_{1,0} & = 1.0 \\
C_t & = 1.0 \\
C_i & = 1.0
\end{align*}
\]

$F_v' = 150 (1.0) = 150 \text{ psi}$
Analysis Example

Check that actual ≤ allowable
- \( f_b \leq F'_b \)
- \( f_v \leq F'_v \)

Check deflection
Check bearing (\( F_{cr} \geq \text{Reaction} / A_{bearing} \))

Design Procedure

**Given:** load, wood, span

**Req'd:** member size

1. **Find Max Shear & Moment**
   - Simple case – equations
   - Complex case - diagrams

2. **Determine allowable stresses**

3. **Solve** \( S = M / F'_b \)

4. **Choose a section from Table 1B**
   - Revise DL and \( F_b \)

5. **Check shear stress**
   - First for \( V_{max} \) (easier)
   - If that fails try \( V \) at \( d \) distance from support.
   - If the section still fails, choose a new section with \( A = 1.5V / F'_v \)

6. **Check deflection**

7. **Check bearing**
Design Example

Given: load, wood, span
Req'd: member size

1. Find Max Shear & Moment
   • Simple case – equations
   • Complex case - diagrams

2. Determine allowable stresses
   (given in this example)
   \[ F'_b = 1000 \text{ psi} \]
   \[ F'_v = 100 \text{ psi} \]

3. Solve \( S = \frac{M}{F'_b} \)

4. Choose a section from \( S \) table
   • Revise DL and \( F'_b \)

5. Check shear stress
   • First for \( V \) max (easier)
   • If that fails try \( V \) at \( d \) distance
     (remove load \( d \) from support)
   • If the section still fails, choose a
     new section with \( A = 1.5V/F'_v \)

6. Check deflection

7. Check bearing
Design Example

Given: load, wood, span  
Req'd: member size

4. Wood Beam Design

Design a 2x dimensioned lumber floor joist to carry the given dead + live floor load. Assume the floor meets conditions of 4.4.1 (span CL=11.0), Also, C1, C2U, and CL = 1.0. Find the short term deflection of your chosen beam under live load only (100% LL is sustained). Compare your LL deflection with the code limit of L/360.

<table>
<thead>
<tr>
<th>DATASET: 1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood Species</td>
<td>HEM-FIR</td>
<td></td>
</tr>
<tr>
<td>Wood Grade</td>
<td>No.1</td>
<td></td>
</tr>
<tr>
<td>Span</td>
<td>20 FT</td>
<td></td>
</tr>
<tr>
<td>Joist Spacing, o.c.</td>
<td>12 IN</td>
<td></td>
</tr>
<tr>
<td>Moisture Content, m.c.</td>
<td>15 %</td>
<td></td>
</tr>
<tr>
<td>Floor DL</td>
<td>7 PSF</td>
<td></td>
</tr>
<tr>
<td>Floor LL</td>
<td>35 PSF</td>
<td></td>
</tr>
</tbody>
</table>

Determine allowable stresses

- $F_b$ and $F_v$ (from NDS)

### Table 4A

**Reference Design Values for Visually Graded Dimension Lumber
(2" - 4" thick)\(^1,2,3\)**

(All species except Southern Pine — see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

<table>
<thead>
<tr>
<th>Species and commercial grade</th>
<th>Size classification</th>
<th>Bending</th>
<th>Tension parallel to grain</th>
<th>Shear parallel to grain</th>
<th>Compression perpendicular to grain</th>
<th>Compression parallel to grain</th>
<th>Modulus of Elasticity</th>
<th>Specific Gravity</th>
<th>Grading System</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEM-FIR</td>
<td>Select Structural</td>
<td>1,400</td>
<td>925</td>
<td>150</td>
<td>405</td>
<td>1,500</td>
<td>1,500,000</td>
<td>580,000</td>
<td>WCLIB WWPA</td>
</tr>
<tr>
<td>No. 1 &amp; Btr</td>
<td>2&quot; &amp; wider</td>
<td>1,200</td>
<td>725</td>
<td>150</td>
<td>405</td>
<td>1,500</td>
<td>1,500,000</td>
<td>550,000</td>
<td></td>
</tr>
<tr>
<td>No. 2</td>
<td>2&quot; &amp; wider</td>
<td>850</td>
<td>525</td>
<td>150</td>
<td>405</td>
<td>1,500</td>
<td>1,500,000</td>
<td>470,000</td>
<td></td>
</tr>
<tr>
<td>No. 3</td>
<td>2&quot; &amp; wider</td>
<td>450</td>
<td>300</td>
<td>150</td>
<td>405</td>
<td>725</td>
<td>1,200,000</td>
<td>440,000</td>
<td></td>
</tr>
<tr>
<td>Stud</td>
<td>2&quot; &amp; wider</td>
<td>675</td>
<td>400</td>
<td>150</td>
<td>405</td>
<td>800</td>
<td>1,200,000</td>
<td>470,000</td>
<td></td>
</tr>
<tr>
<td>Construction</td>
<td>2&quot; - .4&quot; wide</td>
<td>975</td>
<td>600</td>
<td>150</td>
<td>405</td>
<td>1,550</td>
<td>1,300,000</td>
<td>470,000</td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td>2&quot; - .4&quot; wide</td>
<td>550</td>
<td>325</td>
<td>150</td>
<td>405</td>
<td>1,300</td>
<td>1,200,000</td>
<td>440,000</td>
<td></td>
</tr>
<tr>
<td>Utility</td>
<td>2&quot; - .4&quot; wide</td>
<td>250</td>
<td>150</td>
<td>150</td>
<td>405</td>
<td>850</td>
<td>1,100,000</td>
<td>400,000</td>
<td></td>
</tr>
</tbody>
</table>
Design Example

Determine allowable stresses.

Since the size is not known you have to skip $C_F$ (or make a guess).

$$F_b = F'_b (\cos \alpha)$$

$$= 975 \left(1.0 \times 1.15 \times 1.0 \times C_F \right) \approx 1121 \text{ psi}$$

$$\frac{C_D}{C_F}$$

$$F'_V = F_V \left(C_D, C_M, C_e, C_I \right)$$

$$= 150 \left(1.0 \times 1.0 \times 1.0 \times 1.0 \right) = 150 \text{ psi}$$
Design Example

Determine Moment from Loading

First find the uniform beam load, \( w \), from the floor loading.

\[
w = \left( \frac{psf}{12} \right) \frac{0.5C}{12} = \frac{FLF}{42}
\]

\[
(7 + 35) \frac{12}{12} = 42 \text{ psf}
\]

With the beam loading, calculate the maximum moment.

\[
M = \frac{wL^2}{8} = \frac{42 (20)^2}{8} = 2100 \text{ in}^2
\]

Design Example

Estimate the Required Section Modulus.

\[
S_x = \frac{M}{F_b} = \frac{2100 (12)}{1121} = 22.47 \text{ in}^3
\]

Compare this required \( S_x \) to the actual \( S_x \) of available sections in NDS Table 1B. Remember CF will be multiplied which may make some pass which at first fail.

From Table 1B (NDS):

\[
\begin{align*}
2 \times 10 & \quad 21.39 \quad (C_F = 1.1) \quad \text{MIGHT WORK} \\
2 \times 12 & \quad 31.64 \quad (C_F = 1.0)
\end{align*}
\]
Design Example

Choose a section and test it (by analysis with all factors including $C_F$)

\[ \text{TRY } 2 \times 10 \quad C_F = 1.1 \]

\[ F_b = 975 \left( \frac{115}{1.1} \right) = 1233.3 \text{ psi} \]

\[ f_b = \frac{F_b}{A} = \frac{1178}{21.39} = 54.84 \text{ psi; } < 1233 \text{ psi; OK} \]

\[ f_v = \frac{3V}{2A} = \frac{16.5 \left( \frac{420}{13.88} \right)}{13.88} = 45.39 \text{ psi; } < 160 \text{ psi; OK} \]

\[ \therefore \text{ USE } 2 \times 10 \]

Design Example

Check Deflection

In this case LL only against code limit of L/360

\[ LL = 35 \text{ psi} = 35 \text{ plf} \]

\[ \Delta_{LL} = \frac{5wL^4}{384EI} = \frac{5(35)(20)^4(1728)}{384(1500000)(98.93)} = 0.849'' \]

\[ \Delta_{\text{LIMIT}} = \frac{L}{360} = \frac{20(12)}{360} = 0.560'' \]

\[ 0.849'' > 0.560'' \because \text{FAILS} \]