Steel Column Analysis and Design

- Failure Modes
- Effects of Slenderness
- Stress Analysis of Steel Columns
- Capacity Analysis of Steel Columns
- Design of Steel Columns

Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

\[ P_{cr} = \frac{\pi^2 AE}{(KL/r)^2} = \frac{\pi^2 IE}{KL^2} \]

\[ r = \sqrt{\frac{I}{A}} \]

\[ I = Ar^2 \]

- A = Cross sectional area (in²)
- E = Modulus of elasticity of the material (lb/in²)
- K = Stiffness (curvature mode) factor
- L = Column length between pinned ends (in.)
- r = radius of gyration (in.)

\[ f_{cr} = \frac{\pi^2 E}{(KL/r)^2} \leq F_{cr} \]
Analysis of Steel Columns

Conditions of an Ideal Column

- initially straight
- axially loaded
- uniform stress (no residual stress)
- uniform material (no holes)
- no transverse load
- pinned (or defined) end conditions

Analysis of Steel Columns

**Short columns**
Fail by material crushing
Plastic behavior

**Intermediate columns**
Crush partially and then buckle
Inelastic behavior
Local buckling – flange or web
Flexural torsional buckling - twisting

**Long columns**
Fail in Euler buckling
Elastic behavior

![Diagram of slenderness formula](image)
Analysis of Steel Columns

Estimate of effective length factor, \( K \)

\[ KL = L \]

\[ K = 1.0 \quad K = 0.50 \quad K = 0.70 \]

Column is in this position after sideways and joint rotation

\( K = 2.0 \)

<table>
<thead>
<tr>
<th>TABLE C-A-7.1</th>
<th>Approximate Values of Effective Length Factor, ( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>Buckled shape of column is shown by dashed line</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Theoretical ( K ) value</th>
<th>Recommended design conditions</th>
<th>End condition code</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.66</td>
<td>Rotation fixed and translation fixed</td>
</tr>
<tr>
<td>0.7</td>
<td>0.70</td>
<td>Rotation free and translation fixed</td>
</tr>
<tr>
<td>1.0</td>
<td>1.00</td>
<td>Rotation fixed and translation free</td>
</tr>
<tr>
<td>1.0</td>
<td>1.2</td>
<td>Rotation free and translation free</td>
</tr>
<tr>
<td>2.0</td>
<td>2.00</td>
<td>Rotation fixed and translation fixed</td>
</tr>
<tr>
<td>2.0</td>
<td>2.1</td>
<td>Rotation free and translation fixed</td>
</tr>
<tr>
<td>2.0</td>
<td>2.00</td>
<td>Rotation fixed and translation fixed</td>
</tr>
</tbody>
</table>

University of Michigan, TCAUP
Structures II
Slide 6 of 22
Determining K factors by Alignment Charts

Sidesway Inhibited:
Braced frame
1.0 > K > 0.5

Sidesway Uninhibited:
Un-braced frame
unstable > K > 1.0

More Pinned:
If \( \frac{I_c}{L_c} \) is large
and \( \frac{I_g}{L_g} \) is small
The connection is more pinned

More Fixed:
If \( \frac{I_c}{L_c} \) is small
and \( \frac{I_g}{L_g} \) is large
The connection is more fixed

\[ G = \frac{\sum (EI/L)_{column}}{\sum (EI/L)_{beam}} \]
Analysis of Steel Columns - LRFD

Euler equation:

\[ F_e = \frac{\pi^2 E}{(KL)^2} \]

Short & Intermediate Columns:

\[ F_{cr} = 0.658 \frac{F_y}{F_y} \]

Long Columns:

\[ F_{cr} = 0.877 F_e \]

Analysis of Steel Columns
**pass / fail** by LRFD

Data:
- Column – size, length
- Support conditions
- Material properties – Fy
- Factored load – Pu

Required:
- Pu \( \leq \) \( \varnothing \) Pn (pass)

1. Calculate slenderness ratios. \( Lc/r \), \( Lc=KL \)
   The largest ratio governs.

2. Check slenderness ratio against upper limit of 200 (recommended)

3. Calculate transition slenderness \( 4.71 \sqrt{E/F_y} \)
   and determine column type (short or long)

4. Calculate \( F_{cr} \) based on slenderness

5. Determine \( \varnothing Pn \) and compare to Pu
   \( P_n = F_{cr} A_g \phi_p \) \( \varnothing = 0.9 \)

6. If Pu \( \leq \) \( \varnothing \) Pn, then OK
Analysis of Steel Columns
pass / fail by ASD

Data:
• Column – size, length
• Support conditions
• Material properties – Fy
• Factored Load – Pu

Required:
• Pu ≤ ϕPn (pass)

1. Calculate slenderness ratios.
   The largest ratio governs.

2. Check slenderness ratio against upper limit of 200 (recommended)

3. Calculate transition slenderness 
   \( \frac{4.71 \sqrt{E/F_y}}{} \) and determine column type (short or long)

4. Calculate \( F_{cr} \) based on slenderness

5. Determine \( \phi Pn \) and compare to Pu

6. If Pu ≤ \( \phi Pn \), then OK
Analysis of Steel Columns

capacity by LRFD

Data:
- Column – size, length
- Support conditions
- Material properties – Fy

Required:
- Max load capacity

1. Calculate slenderness ratios. The largest ratio governs. \( r_x \) or \( r_y \)
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness \( 4.71 \sqrt{\frac{E}{F_y}} \) and determine column type (short or long)
4. Calculate \( F_{cr} \) based on slenderness
5. Determine \( \phi P_n \) and Compute allowable capacity:
   \[ P_u = \phi P_n = \phi F_{cr} \Delta_\text{d} \]

Capacity Example 1

Free standing column
Third floor studio space
Supports roof load = 20 psf DL + SL
   snow \( \approx 15 \text{lbs} \) / FT depth
Capacity Example 1


2. Check slenderness ratio against upper limit of 200 (recommended)

3. Calculate transition slenderness $4.71\sqrt{E/F_y}$ and determine column type (short or long)

4. Calculate $F_{cr}$ based on slenderness

5. Determine $\phi P_n$ and Compute allowable capacity: $P_u = \phi P_n$
### Capacity Example 2
long column – using equations

\[
\begin{align*}
rx & = 3.51 \text{ in.} \\
\rightarrow ry & = 2.03 \text{ in.}
\end{align*}
\]

#### Table G1
Buckling Length Coefficients, \( K_o \)

<table>
<thead>
<tr>
<th>Buckling mode</th>
<th>Theoretical ( K_o ) value</th>
<th>Recommended ( K_o ) when ideal conditions approximated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation fixed, translation fixed</td>
<td>0.5</td>
<td>0.65</td>
</tr>
<tr>
<td>Rotation free, translation fixed</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Rotation free, translation free</td>
<td>2.0</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Slenderness \( \gamma_y \)

\[
\frac{\gamma_y}{K_o} = \frac{0.8(25)}{2.03} = 11.82
\]

Euler Buckling

\[
F_\text{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{972,200}{20.47} = 20.47 \text{ kpsi}
\]

Long Column Equation

\[
F_\text{cr} = 0.877 \left( \frac{20.47}{11.82} \right) = 17.95 \text{ kpsi}
\]

Column strength

\[
\phi = \frac{F_{\text{cr}}}{F_\text{y}} = 0.9 \left( \frac{17.95}{10.3} \right) = 1.66 \text{ kpsi}
\]
Design of Steel Columns with AISC Strength Tables

Data:
- Column – length
- Support conditions
- Material properties – $F_y$
- Applied load - $P_{actual}$

Required:
- Column Size

1. Enter table with height, $KL = L_c$
2. Read allowable load for each section to find the smallest adequate size.
3. Tables assume weak axis buckling. If the strong axis controls the length must be divided by the ratio $r_x/r_y$
4. Values stop in table (black line) at slenderness limit, $KL/r = 200$

AISC Critical Stress Table

For previous example $KL/r_y = 118.2$
AISC Critical Stress Table

for previous example $Kl/r_y = 118.2$

To find capacity:

$$\phi P_{cr} = 16.2 \text{ ksi}$$

$$P_u = P_{cr} = \phi P_{cr} A_g$$

$$P_u = 16.2 \times (1.03) = 16.6 \text{ ksi}$$

Steel Frame Construction