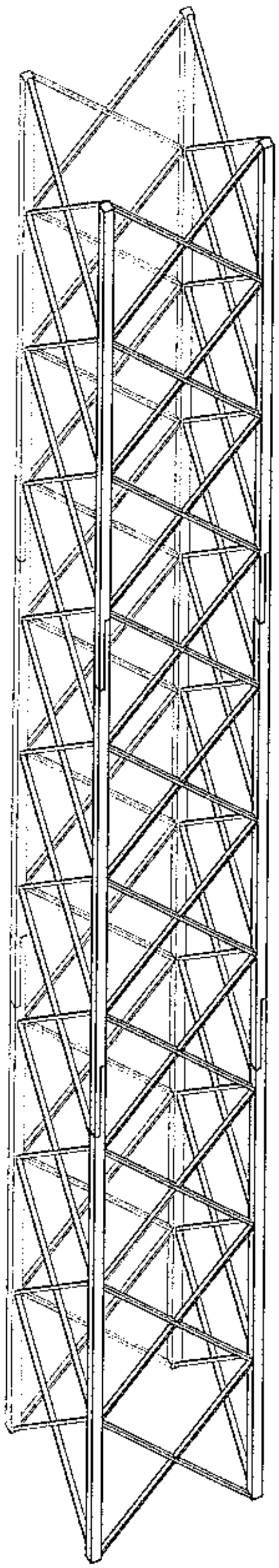
Tower Project: Final Report



59/60

Paul Ligeti & Yinying Chen
"Tower Group"
Structures II
03/28/2016

Preliminary Design Development

Our first design of the tower consisted of a 4" * 4" cross-section, made of 4 columns of 1/4" * 1/4" basswood, and diagonal bracings of 1/8" * 1/8" basswood. We chose to use the aforementioned 1/4" * 1/4" cross-sectional area for our columns because we expected them to bear the brunt of the weight, being the members that ultimately transferred the force of the load to the ground, and thus wanted our columns to be as strong and thick as we were allowed to make them (maximum allowable size was 1/4" * 1/4"). This, consequently, made our columns easily exceed the 25-lb per column minimum load requirement.

We chose to space out our diagonal bracings every 5 inches to ensure that we had enough bracings (10) to support each column, while not having too many that the tower would exceed the weight limit of 4 ounces. Due to weight restrictions, these bracings were ultimately 1/8" * 1/8", allowing us to theoretically craft a tower that did not exceed 4 ounces. The left and right components of the "x" that formed from each bracing did not overlap (in contrast with our final tower design), for the purpose of minimizing weight. However, we ultimately altered that design. One thing that was constant, however, was that the bracings created a tower that was (conceptually) solely composed of triangles, allowing for maximum rigidity and resistance to moment at each joint. This is why we chose a bracing method - it maximized strength, rigidity, and the uniform distribution of load, while minimizing weight.

+3 +3

We chose to us a square design because it would increase the tower's rigidity and stiffness, would be easy and accurate to craft (and thus avoid any potential weaknesses from struggling to match angles on a triangular, pentagonal, etc. tower), and would distribute the axial loads uniformly throughout the tower. We did consider constructing a tower with a triangular cross-section to eliminate weight, but decided against it based on crafting issues, and the idea that the load strength that an extra column added outweighed the rigidity that a triangular tower would innately have.

+3

Ultimately, we made only 2 changed to our tower. First, instead of having each component of the bracing meet at a t-joint, we instead had them overlap. We changed this design because it would make the craft more accurate, and would allow for a larger surface area overlap. This surface area overlap increased the strength of the tower by increasing the areas connected by the wood glue that we were using. In addition, the glue we were using was supposedly stronger than the material strength of the wood itself, in certain connections.

+3

See Figure 1 for visual clarity.

+4

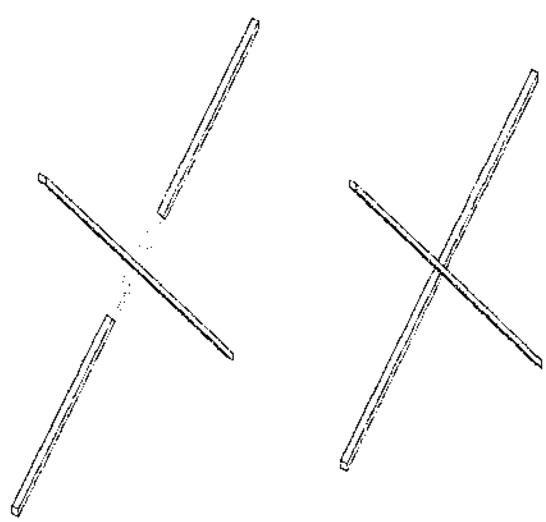


Fig. 1: T-joint (left) vs overlap (right)

Preliminary Design Development (continued)

The second change was in the nature of constructing each of the 4 columns. Originally, we had planned to end-join each component, as we could find no single piece of basswood that was 50 inches in length (24 inches was the maximum, so we were orignally planning on end-joining 3 pieces of 50/3 = 16.67" long basswood pieces together to create each 50" piece). However, we also knew that the glue we were using was theoretically stronger than the basswood material itself, in certain connections. Thus, we created a notched connection (a 1" * 1/8" * 1/4" chunk was taken out of each component, allowing for a symmetrical overlap) between each of the 3 16.667" pieces that maximized surface area, allowing for a maximum application of the aforementioned wood glue.

See figure 2 for visual clarity.

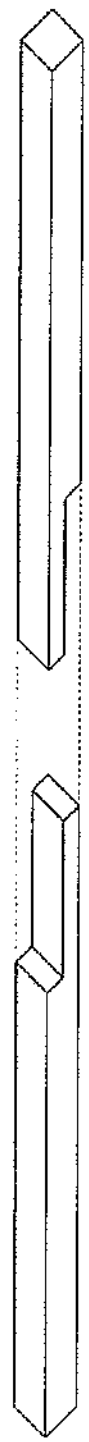


Fig. 2: Notched Connection

Revised/Tested Tower Design Analysis

**Note: We are using the same equations and constants as in our preliminary report. Although we did change the overlap in the bracing, and the method of connecting each 16.66" piece of each of the 4 columns, this does not affect any of the variables in the slenderness ratio, radius of gyration, buckling, and crushing equations. Rather, it operates on the same principal as in the preliminary report: that the overlaps and joints between pieces are substituted by a conceptual tower made of only 1 piece of wood (not multiple segments), which has no grain or other connection and material points of weakness, and is crafted perfectly.*

Derivation of cross-sectional areas of each member:

Like in our original report, our target capacity was *not* 50 lb; rather, we aimed for the same 200 lb as when we made our preliminary report. We can solve for the required slenderness ratio using the Euler buckling equation...

$$KL/r=\sqrt{(\pi^2EA/P)}= \sqrt{(\pi^2*1{,}650{,}000*0.625/50)}=142.67$$

...where "P" is 200/4 columns = 50 lb per column. This is a very high slenderness ratio (the higher the ratio, the easier it is to attain), and thus it is very feasible. Next, we assume that K = 1.0, and calculate the radius of gyration, r...

$$r=KL/142.67=1*5/142.67=0.035$$

...where "5" indicated the spacing between each bracing member (and thus the calculable height of each vertical segment), in inches. Next, we calculate whether the member we are using is appropriate for the weight...

$$r=h/\sqrt{12}\ h=0.035*\sqrt{12}=0.12 \text{ in.}$$

.12 in is smaller than .71268 in (the r for a 1/4"*1/4" basswood column, gathered from the table of basswood cross-sectional properties provided online). This affirms the viability of our choice of the 1/4"*1/4" basswood column. Next, we calculate the crushing capacity of each column...

$$P=F*A=4745*0.0625=296.5625\text{lbs}$$

+2

This is significantly larger than our crushing capacity of 50 pounds per column. Given that our bracing is so light, we felt that increasing the cross-sectional area of each column to the maximum allowable area of 1/4" * 1/4" was an effective way to increase strength without increasing weight too much.

Predicted weight estimate of the entire tower:

Basswood: 20 lb/ft^3, divided by (12^3) = .0116 lb/in^3, multipled by 16 = **.185 oz/in^3**

Total weight: [.185*({4 vertical members * 50in * .25in * .25 in} + {80 diagonal members * 6.4in * .125in * .125in})] + [.25 oz of glue)] =**4.04 oz.**

Of course, the actual tower tower (at 4.1 ounces) did end up exceeding the allowable weight by one ounce, and thus our estimate by .06 ounces. This can be explained by the probible case that we did end up using more glue than we expected; rather than .25 ounces, we used .31 ounces, or 124% of our predicted glue.

This runs in contrast with our goal to reduce the amount of glue used by .04 ounces. Then again, less glue would also likely mean less strength.

+2

Predicted Capacity...

**Note: we first determine the capacity of vertical members, as they are the crucial components of our tower. This is because they are expected to transfer all of the load into the ground; the bracings do not do this. Rather, what the diagonal bracings do is uniformly distribute the load throughout the tower.*

...Of vertical members:

Length = 50"/10 spaces = **5"**

Area = **.0625 in^2**

r = √(I/A) = ... width/√12 <== Based off of JY recitation notes on wood columns
= .25/√12
= .07217

Vertical crushing: $P = F_c A = (4745\ psi)(.0625 in^2)$ = **296.56 lb**

Vertical buckling: P = $(\pi^2)AE/(KL/r)^2$
= $1017802.95/((1*5)/.07217)^2$
= $1017802.95/(65.252)^2$
= **212.05 lb**

*This is spread out over 4 members, making the buckling capacity a whopping **848** lbs. Again, this assumes perfect craft, no wood grain or other material factors, perfect environmental conditions, etc. Thus, our equations point to crushing occuring before buckling. However, as detailed in the "Testing Results" portion of this report, it was in fact buckling that occured first. This may be explained by a combination of imperfect craft, material properties, and imperfect environmental conditions.

...Of the tower as whole (as if it were one uniform piece, not multiple segments):

Moment of interia (See figure 3 for visual clarity):

Using the subtractive method (subtracting void of "column" from 4" * 4" occupied area of "column":

$I = \Sigma I_{total} - I_{void}$

$I = [((4in)(4in)^3)/12] - [2*((.25in)(3.5in)^3)/12] - [((3.5in)(4in)^3)/12]$

I = **.88 in^4**

Figure 3:

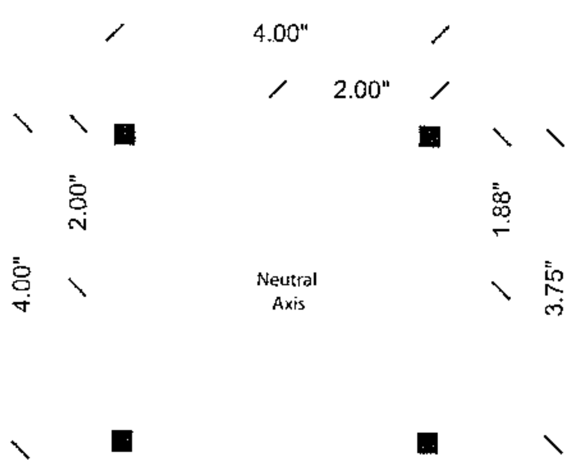


Fig. 3: Interia Diagram

Predicted Capacity (continued)

Critical buckling load of the tower as one cohesive piece:

Assume K = 1

Radius of gyration r = √(.88in⁴/(4*.0625in²)) = **1.876 in**

Slenderness ratio KL/R = 1(50in)/1.876 = **26.6524**

$F_{cr} = (\pi^2)(1{,}650{,}000\text{psi})/(26.6542)^2 =$ **22,921.95 psi**

+2

$P_{cr} = F_{cr}A = (22{,}921.95\text{ psi})(4*.0625\text{in}^2) =$ **5,730.4875 lb**

Of course, this is assuming that the tower is one large column with perfect craft. Our tower was not, and could never be, a tower of one homogenous piece, let alone a homogenouspiece with no grain or other material properties inherently found in wood. Thus, it cannot be used as an accurate estimation of our tower's weight capacity; rather, we use the previous calculations which gave us 296.56 and 848 lbs for crusing and buckling, respectively. Of course, even these equations assume perfect internal and external conditions. As we detail in the "Testing Results" section of this report, we fell short (as we expected) of these weight limits.

+2

Properties of Basswood:

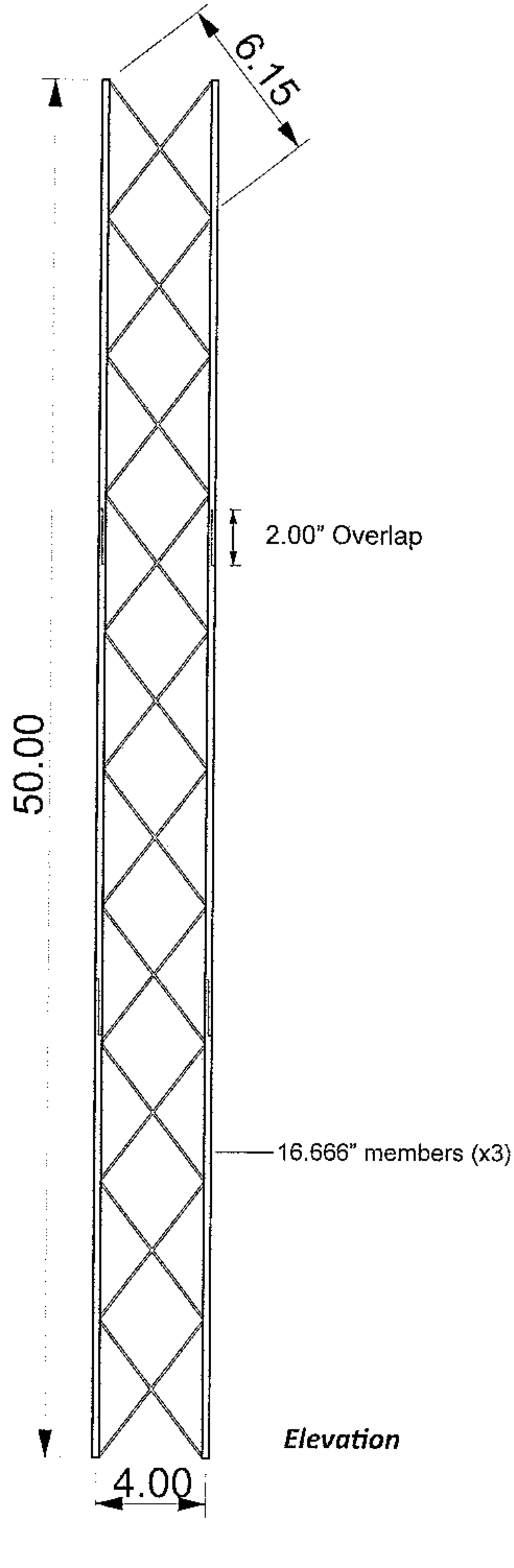
Density (oven dry)	20 pcf *
E (buckling)	1,650,000 psi **
F (Compression ∥ to grain)	4745 psi *
F (Compression ⊥ to grain)	377 psi *
F (Tension ∥ to grain)	4500 psi (estimate)
F (Tension ⊥ to grain)	348 psi *
F (Shear ∥ to grain)	986 psi *
F (Flexure)	5900 psi *

* from http://www.matweb.com/ ** tested by PvB (small pieces in compression)

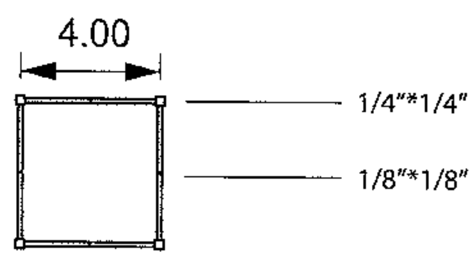
not the right table
need a table of each member +1

Fig. 4: Basswood properties

Illustrations



Elevation

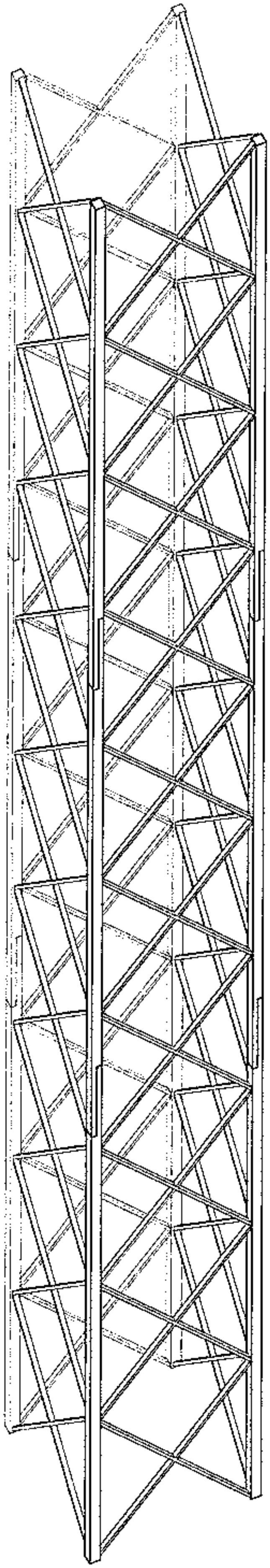


Cross-Section

Illustrations (continued)

**Note: Dimensions labelled on plan/elevation*

Isometric



+8

Testing Results/Post-Testing Analysis

Final weight of tower: 4.1 oz
Final height of tower: 50 in
Tested capacity of tower: 230 lbs

The tower continued to hold steady and stand up straight until the 200-pound mark. We had been placing weights on the tower in pairs (so 10 pounds at a time), and right before we got the tower to 230 pounds, it began to lean towards the bench, to the right (facing the bench from the camera). After placing the final 10 pounds, the tower leaned significantly more and snapped - all within a very fast timeframe of less than half a second.

As shown in the picture below (figures 5 and 6), the tower buckled outwards towards the left (facing the bench from the camera), and inwards on the right side. What likely happened is that the back right column bore more than 1/4 of the weight - perhaps due to brick placement, perhaps due to craft or material deficiencies - and snapped prematurely as a consequence - it had reached its critical buckling load (*not* critical crushing, as we had expected)! Once that column was broken, the rest inevitably fell because now *they* had to split the 230 pounds evenly, as well as deal with bending and twisting.

More specifically, the column snapped at the intersection of one of the notched connections of the back right column. This makes it likely that the main reason for buckling was both craft and the inherent nature of our notched connection.

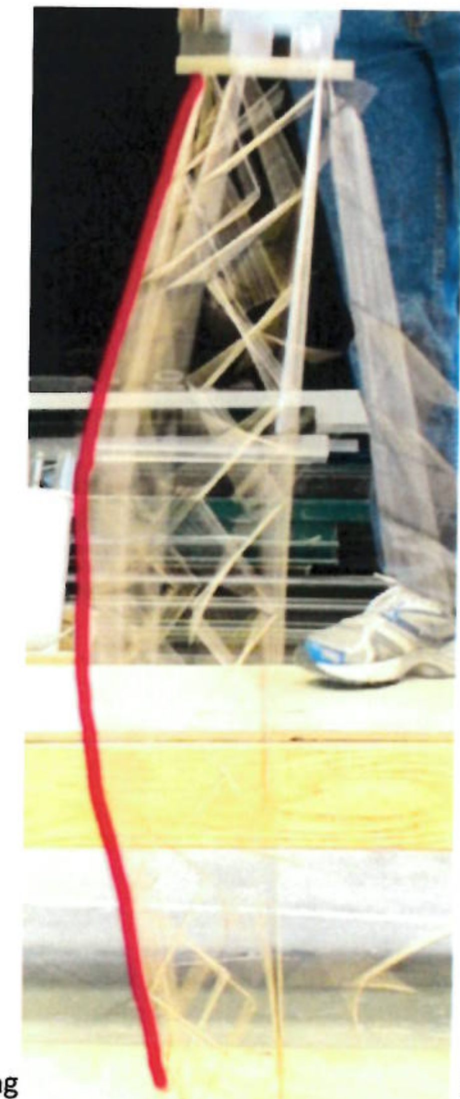


Fig. 5: Outward Buckling

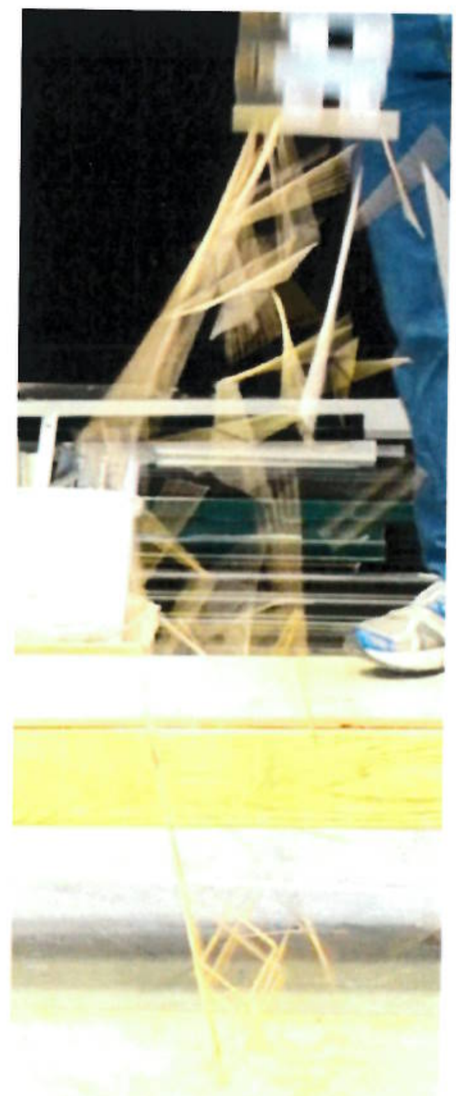


Fig. 6: Collapsing

Post-Testing Analysis

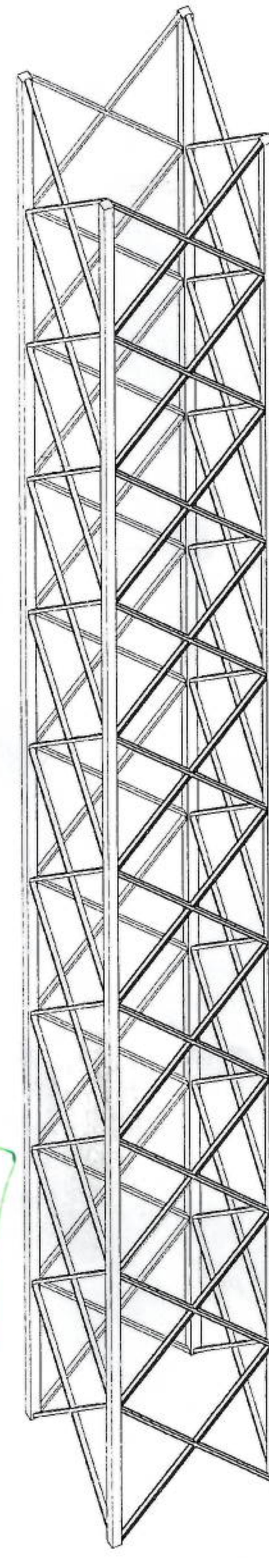
So why did we not meet our 848-lb goal? Due to the inevitable imperfections in craft, joints (both bracing and column notched connections), brick placement, material deficiencies (warping, knotting, etc.), and properties such as wood grain - which determine the integrity of the wood in certain axes - the tower did not hold the weight we expected. In fact, these properties make it incredibly likely that even under perfect environmental conditions - no humidity, a level ground, etc. - the 212 lb/column buckling capacity would have been impossible to achieve in any case.Rather, it held 230 pounds (which was still a significant amount, at 78% of the expected 296.56-lb crushing capacity)! In addition, these aforementioned factors, the tower ended up buckling, not crushing.

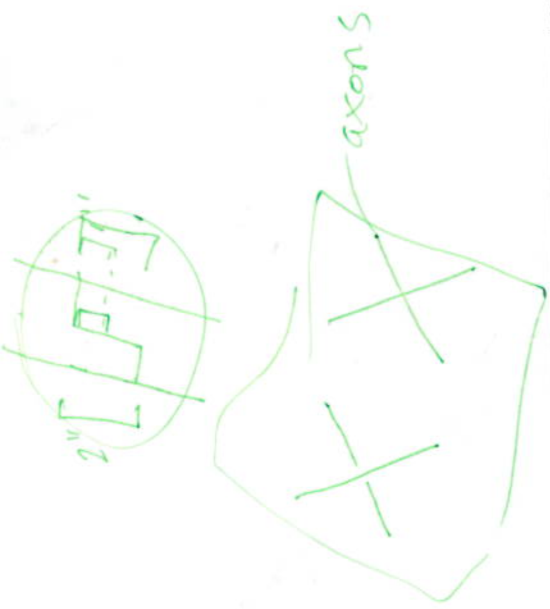
+2 +2

For future improvement, we could aim to make the aforementioned notched column connection stronger - either through a different methof of joining the three components of each column together, or additional support around the connection (such as a wrapping). Also the way the tower leaned suggests that there was an inbalance between the 4 columns, which caused one to bear more of the load. If we align all the columns better, it will carry more load.

+6

Tower Project: Preliminary Report





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Structures II
01/31/2016

Explanation:

We started by looking at precedents - both experimental and real-life implementations. Previous winning groups - Beam Me Up Scotty, Tower 2015, Take a Pisa My Heart - all seemed to use the same method: four long members supported by diagonal bracing and (except for Beam Me Up Scotty) horizontal bracing members. These members serve an important purpose: they shorten the effective buckling length of the four vertical members.

Radio towers use a similar method: vertical supporting masts, supported by diagonal bracing (and guy wires). While these do not experience compression outside of weight, they do experience a lot of wind - lateral forces - that can lead to buckling if not designed correctly.

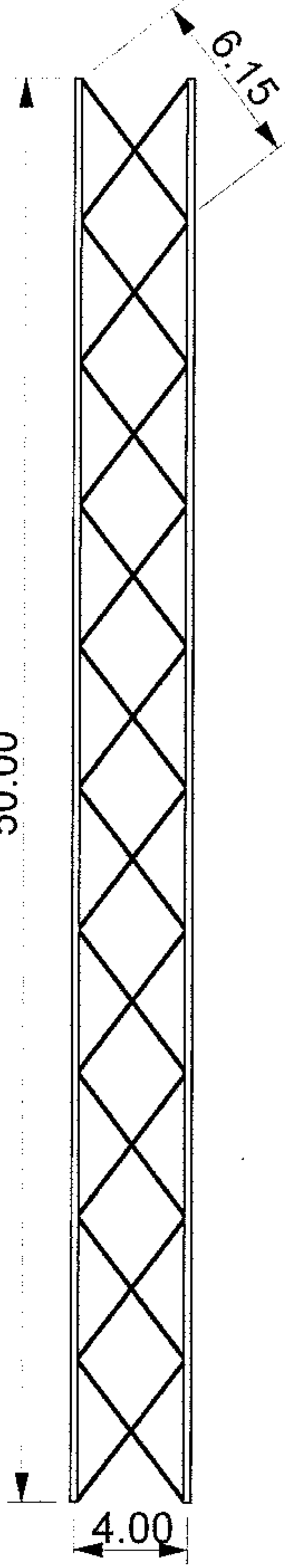
Finally, the optimization sheet given to us also shows similar structures with diagonal bracing. So it was clear that this was the way to go.

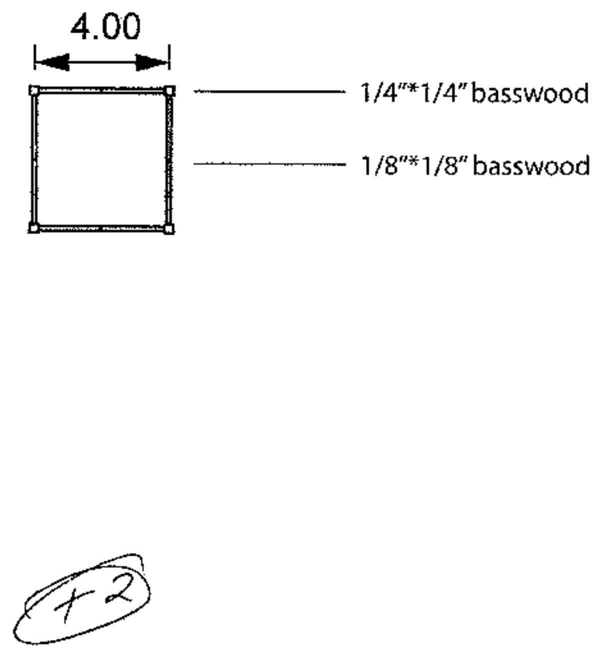
We chose to divide the tower into 10 “levels,” one for every 5 inches. We believe (based on the equations) that this should effectively combat buckling, and in an ideal situation with perfect craft, the tower may be able to support up to **296** pounds!

+2

overall clarity +1

Illustration (Elevation/Cross-Section):





Analysis:

Derivation of cross-sectional areas of each member:

We began our design by aiming at the capacity of 200 lb, *not* 50 lb. Using the Euler buckling equation, we can solve for a required slenderness ratio:

$KL/r=\sqrt{(\pi^2EA/P)}= \sqrt{(\pi^2*1{,}650{,}000*0.625/50)}=142.67$

This is a very high slenderness ratio, and it is very attainable.
Assuming K=1.0, we can get the radius of gyration, r:

$r=KL/142.67=1*5/142.67=0.035$

Checking whether the member we are using is appropriate:

$r=h/\sqrt{12}$ $h=0.035*\sqrt{12}=0.12$ in.

.12 in is smaller than .71268 in (the r for a 1/4"*1/4" basswood column), which confirms the viability of our choice of the 1/4"*1/4" basswood column.

Comparing crushing:

P=F*A=4745*0.0625=296.5625lbs

Compared to our load of 50lbs for each vertical member, this crushing capacity is larger. Thus, our colums should be able to hold up.

+2

Predicted weight estimate of entire tower:

Basswood: 20 lb/ft^3, divided by (12^3) = .0116 lb/in^3, multipled by 16 =

.185 oz/in^3

Total weight: [.185*({4 vertical members * 50in * .25in * .25 in}
+ {80 diagonal members * 6.4in * .125in * .125in})] + [.25 oz of glue)] =

4.04 oz. (can be adjusted) +1

Just keep an eye on it and you should be ok.

Predicted Capacity...

...Of vertical members:

Length = 50"/10 spaces = 5"; Area = .0625"

$r = \sqrt{I/A}$ = ... width/$\sqrt{12}$ <== Based off of JY recitation notes on wood columns
= .25/$\sqrt{12}$
= .07217

Vertical crushing: $P = F_c A = (4745\ psi)(.0625 in^2)$ = **296.56 lb**

Vertical buckling: $P = (\pi^2)AE/(KL/r)^2$
$= 1017802.95/((1*5)/.07217)^2$
$= 1017802.95/(65.252)^2$
= **212.05 lb**

*This is spread out over 4 members, making the buckling capacity a whopping **848** lbs (of course, assuming perfect craft and materials, and no other factors). Thus, crushing will probably happen first.

...Of the tower as whole:

Moment of interia:

Using the subtractive method (subtracting void of "column" from 4" * 4" occupied area of "column":

$I = \Sigma I_{total} - I_{void}$

$I = [((4in)(4in)^3)/12] - [2*((.25in)(3.5in)^3)/12] - [((3.5in)(4in)^3)/12]$

$I = .88\ in^4$

Critical buckling load:

+2

Assume K = 1

$r = \sqrt{(.88in^4/(4*.0625in^2))}$ = 1.876 in

KL/R = 1(50in)/1.876 = 26.6524

$F_{cr} = (\pi^2)(1{,}650{,}000psi)/(26.6542)^2$ = 22,921.95 psi

$P_{cr} = F_{cr}A$ = (22,921.95 psi)($4*.0625in^2$) = **5,730.4875 lb**

Of course, this is assuming that the tower is one large column with perfect craft. One can only dream...