Combined Stress

- Axial vs. Eccentric Load
- Combined Stress
- Interaction Formulas

Axial Stress

- Loads pass through the centroid of the section, i.e. axially loaded
- Member is straight
- Load is less than buckling load

\[ f = \frac{P}{A} \]
Eccentric Loads

- Load is offset from centroid
- Bending Moment = $P \cdot e$
- Total load = $P + M$

Interaction formula

\[ f = \frac{P}{A} + \frac{Mc}{I} \]

Combined Stress

- Stresses combine by superposition
- Values add or subtract by sign
Example

1. Determine external reactions

\[ \Sigma H = 0 = -B(6') + 10(8') + 10(16') + 5(24') \]
\[ B = 60k \]

\[ \Sigma V = 0 = A(6') + 10(8') + 10(16') + 5(24') \]
\[ A = 60k \]

\[ \Sigma F_y = 0 = 60 - 60 \]

\[ FB = A \]

\[ 60k \]
\[ 60k \]
\[ 60k \]
\[ 60k \]
\[ 60k \]

\[ \Sigma F_y = 0 = 15 - 10k - 10k - 5k + Bv \]
\[ Bv = 10k \]

Example

2. Determine internal member forces: Axial and Flexural

3. Determine axial and flexural stresses

\[ W14 \times 34 \quad A = 10.0 \text{ in}^2 \]
\[ S_x = 48.4 \text{ ksi} \]

**FORCE:**
- Axial: \[ F_a = \frac{P}{A} = \frac{60k}{10.0} = 6.0 \text{ kips} \]
- Flexural: \[ M = PL/3 = 10k(8') = 60k - \text{kips} \]

**STRESS:**
- Axial: \[ f_a = \frac{F_a}{A} = \frac{60k}{10.0} = 6.0 \text{ ksi} \]
- Flexural: \[ f_x = \frac{M}{S_x} = \frac{60k}{48.4} = 19.75 \text{ ksi} \]
Example

2. Use interaction formula to determine combined stresses at key locations (e.g. extreme fibers)

Second Order Stress
“P Delta Effect”

With larger deflections this can become significant.

1. Eccentric load causes bending moment
2. Bending moment causes deflection, $\Delta$
3. $P \times \Delta$ causes additional moment
Other Examples

Columns with side loading

Moment frames

Trusses loaded on members

Other Examples

Eccentrically loaded columns

Wind load on walls
For the eccentric load $P = 30000$ LBS, find the actual combined stress, $f$, on each side of the column in psi.

Left side ________ psi

Right side ________ psi

Combined Stress in NDS

**Figure 3G** Combined Bending and Axial Tension

**3.9.2 Bending and Axial Compression**

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Combined Stress in NDS
**Tension + Flexure**

**NDS Equations**

**CASE 1.** Tension is critical. Eq. 3.9-1

\[ \frac{f_t}{F'_t} + \frac{f_b}{F_b} \leq 1.0 \]

\[ f_b - f_t \leq 1.0 \quad \frac{F_b}{F_b} \]

**CASE 2.** Flexure is critical. Eq. 3.9-2

\[ \text{No } C_L \]

\[ \text{No } C_V \]

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**3.9.1 Bending and Axial Tension**

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

\[ \frac{f_t}{F'_t} + \frac{f_b}{F_b} \leq 1.0 \quad \text{TENSION CRIT.} \quad (3.9-1) \]

and

\[ \frac{f_b - f_t}{F_b} \leq 1.0 \quad \text{FLEXURE CRIT.} \quad (3.9-2) \]

where:

- \( F'_b \) = reference bending design value multiplied by all applicable adjustment factors except \( C_t \)
- \( F''_b \) = reference bending design value multiplied by all applicable adjustment factors except \( C_v \)
Example Problem

Given: Queen Post truss
Hem-Fir No.1 & Better
F_b = 1100 psi
F_t = 725 psi
F_c = 1350 psi
E_{min} = 550000 psi

span = 30 ft. spaced 48\textquoteright\ o.c.
D + S Load = 44 psf (projected)
D (attic + ceiling) = 8 psf

bottom chord: 2x8
top chord: 2x10

Find: pass/fail

\begin{align*}
\frac{f_t}{F_t} + \frac{f_b}{F_b} &\leq 1.0 \\
\frac{f_b - f_t}{F_b} &\leq 1.0
\end{align*}

1. Determine truss joint loading

Example (cont.)

2. Determine the external end reactions of the whole truss. The geometry and loads are symmetric, so each reaction is \( \frac{1}{2} \) of the total load.

3. Use an FBD of the reaction joint to find the chord forces. Sum the forces horizontal and vertical to find the components.

Top chord = 4.96 k compression
Bottom chord = 4.44 k tension
Example
bottom chord 2x8

4. Calculate the **actual** axial and flexural stress.

\[ f_t = 408.3 \text{ psi} \]
\[ f_b = 821.9 \text{ psi} \]

5. Determine **allowable** stresses using applicable factors:

- **tension:** (D+S)
  \[ F_t' = F_t (C_D \cdot C_F) \]
  \[ F_t' = 725 (1.15 \cdot 1.2) = 1000 \text{ psi} > 408.3 \]

- **flexure:** (D)
  \[ F_b = F_b (C_D \cdot C_L \cdot C_F) \]
  \[ F_b' = 1100 (0.9 \cdot 1.0 \cdot 1.2) = 1188 \text{ psi} > 821.9 \]

- **flexure:** (D+S)
  \[ F_b = F_b (C_D \cdot C_L \cdot C_F) \]
  \[ F_b' = 1100 (1.15 \cdot 1.0 \cdot 1.2) = 1518 \text{ psi} \]

---

Example
bottom chord 2x8

5. Determine **allowable** stresses using applicable factors:

- **tension:** (D+S)
  \[ F_t' = F_t (C_D \cdot C_F) \]
  \[ F_t' = 725 (1.15 \cdot 1.2) = 1000 \text{ psi} > 408.3 \]

- **flexure:** (D)
  \[ F_b = F_b (C_D \cdot C_L \cdot C_F) \]
  \[ F_b' = 1100 (0.9 \cdot 1.0 \cdot 1.2) = 1188 \text{ psi} > 821.9 \]

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  \[ F_b = F_b (C_D \cdot C_L \cdot C_F) \]
  \[ F_b' = 1100 (1.15 \cdot 1.0 \cdot 1.2) = 1518 \text{ psi} \]
3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

\[ \frac{f_t}{F_{t0}} + \frac{f_c}{F_{b0}} \leq 1.0 \]  \hspace{1cm} \text{TENSION CRIT. (3.9-1)}

and

\[ \frac{f_b - f_c}{F_b} \leq 1.0 \]  \hspace{1cm} \text{FLEXURE CRIT. (3.9-2)}

where:

- \( F_{t0} \) = reference bending design value multiplied by all applicable adjustment factors except \( C_t \)
- \( F_{b0} \) = reference bending design value multiplied by all applicable adjustment factors except \( C_b \)

\[(3.9-2) \]

\[ \frac{408.3}{1000} + \frac{821.9}{1518} \]

\[ 0.4083 + 0.5414 = 0.95 \]

\[ 0.95 < 1.0 \] \hspace{1cm} \text{PASS}

\[(3.9-1) \]

\[ \frac{821.9 - 408.3}{1518} \]

\[ 0.2724 \]

\[ 0.27 < 1.0 \] \hspace{1cm} \text{PASS}

3.9.2 Bending + Axial Compression

Members subjected to a combination of bending about one or both principal axes and axial compression (see Figure 3H) shall be so proportioned that:

\[ \left( \frac{f}{F_{e1}} \right)^2 + \left( \frac{f_c}{F_{b2}} \right)^2 \leq 1.0 \]  \hspace{1cm} (3.9-3)

where:

\[ f < F_{e1} = \frac{0.822E_{\text{min}}}{(l_{e1}/d_1)^2} \]  \hspace{1cm} \text{for either uniaxial edgewise bending or biaxial bending}

and

\[ f_c < F_{b2} = \frac{0.822E_{\text{min}}}{(l_{e2}/d_2)^2} \]  \hspace{1cm} \text{for uniaxial flatwise bending or biaxial bending}

and

\[ f_{b1} < F_{b2} = \frac{1.20E_{\text{min}}}{(R_b)^2} \]  \hspace{1cm} \text{for biaxial bending}

\[ f_{b1} = \text{actual edgewise bending stress (bending load applied to narrow face of member)}, \text{ psi} \]

\[ f_{b2} = \text{actual flatwise bending stress (bending load applied to wide face of member)}, \text{ psi} \]

\[ d_1 = \text{wide face dimension (see Figure 3H), in.} \]

\[ d_2 = \text{narrow face dimension (see Figure 3H), in.} \]
Example
top chord 2x10

4. Calculate the actual axial and flexural stress.

\[ f_c = 357.5 \text{ psi} \]
\[ f_b = 694.2 \text{ psi} \]

5. Determine allowable stresses using applicable factors:

(compression: D+S)

\[ F_c' = F_c (C_D \cdot C_F \cdot C_P) \]
\[ F_c' = 1350 \left( 1.15 \cdot 1.0 \cdot 0.897 \right) = 1392.7 \text{ psi} > 357.5 \]

(flexure: D+S)

\[ F_b' = F_b (C_D \cdot C_L \cdot C_F) \]
\[ F_b' = 1100 \left( 1.15 \cdot 1.0 \cdot 1.1 \right) = 1392 \text{ psi} > 694.2 \]

Eq. 3.9-3

\[
\left( \frac{f_c}{F_c'} \right)^2 + \frac{f_{b1}}{F_{b1}' \left[ 1 - \left( \frac{f_c}{F_{c1}} \right) \right]} \leq 1.0
\]

COMP. + FLEXURE X-X

where:

\[ f_c < F_{c1} = \frac{0.822 E_{min}}{(d_{w2} / d_{j})^2} \]

EULER 1 for either uniaxial edge-wise bending or biaxial bending

and

\[ f_c < F_{c2} = \frac{0.822 E_{min}}{(d_{w1} / d_{j})^2} \]

EULER 2 for uniaxial flatwise bending or biaxial bending

and

\[ f_{b2} < F_{b2} = \frac{\sqrt{2} E_{min}}{d_j} \]

LTB for biaxial bending

\[ f_{c1} = \text{actual edgewise bending stress (bending load applied to narrow face of member)} \]
\[ f_{c2} = \text{actual flatwise bending stress (bending load applied to wide face of member)} \]
\[ d_w = \text{wide face dimension (see Figure 3H)} \]
\[ d_j = \text{narrow face dimension (see Figure 3H)} \]

Compressional:

\[ \left( \frac{f_c}{F_{c1}} \right)^2 = \left( \frac{357.5}{1392.7} \right)^2 = 0.0659 \]

Flexure:

\[ \frac{f_{b1}}{F_{b1}'} = \frac{694.2}{1392} = 0.4987 \]

Amplification factor:

\[ \frac{1}{1 - (357.5/3820)} = 0.906 \]

\[ 0.4987 \cdot 1.103 = 0.550 \]

Combination:

\[ 0.0659 + 0.550 = 0.616 \]
\[ 0.616 < 1.0 \text{  Pass} \]
Combined Stress in NDS

procedure

Exterior stud wall under bending + axial compression

1. Determine load per stud
2. Use axial load and moment to find actual stresses $f_c$ and $f_b$
3. Determine load factors
4. Calculate factored stresses
5. Check NDS equations

\[
\left[ \frac{f_c}{F_c'} \right]^2 + \frac{f_b}{F_{b1}' \left[ 1 - \left( \frac{f_c}{F_{c1}} \right) \right]} \leq 1.0 \quad (3.9-3)
\]

and

\[
\frac{f_c}{F_{c2}'} + \left( \frac{f_b}{F_{b1}'} \right)^2 < 1.0 \quad (3.9-4)
\]

Combined Stress in NDS

example

Exterior stud wall under bending + axial compression

1. Determine load per stud
2. Use axial load and moment to find actual stresses $f_c$ and $f_b$
Combined Stress in NDS example

Exterior stud wall under bending + axial compression

3. Determine load factors (bending)

Size Factors, \( C_F \)

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\[ F_b = 775 \text{ psi} \quad F_c = 1000 \text{ psi} \quad E_{shk} = 400000 \text{ psi} \]

\[ C_D = 1.6 \quad (\text{wind}) \]
\[ C_F = 1.5 \quad (\text{for} \ F_b) \quad \sqrt{1.15} (\text{for} \ F_c) \]
\[ C_L = 1.0 \quad (\text{braced by sheathing}) \]
\[ C_P = 1.15 \quad (\leq 24^\circ \text{C}) \]

Combined Stress in NDS example

Exterior stud wall under bending + axial compression

4. Calculate factored stresses

Bending Stress

\[ M = \frac{wL^2}{8} = \frac{26 \times 99.5^2}{8} = 223.4 \text{ in} \cdot \text{lb} \]
\[ S_x = 3.06 \text{ in}^3 \]
\[ f_b = \frac{M}{S_x} = \frac{223.4(12)}{3.06} = 876 \text{ psi} \]
Combined Stress in NDS
example

Exterior stud wall under bending + axial compression

4. Calculate factored stresses

**Bending Stress**

\[
F_b = 775 \text{ psi}
\]

\[
C_D = 1.6 \quad C_F = 1.5
\]

\[
C_M = 1.0 \quad C_{F_0} = 1.0
\]

\[
C_t = 1.0 \quad C_i = 1.0
\]

\[
C_L = 1.0 \quad C_r = 1.15
\]

\[
F_b^1 = 775 \cdot (1.6) \cdot (1.5) \cdot (1.15)
\]

\[
= 2139 \text{ psi}
\]

Combined Stress in NDS
example

Exterior stud wall under bending + axial compression

3. Determine load factors (compression)

\[
P = 900 \text{ kips}
\]

\[
k = 1.0
\]

\[
f = 99.5\text{''}
\]

\[
C_p = \frac{1 + (F_{CE}/F_c^*)}{2c} - \sqrt{\left[1 + (F_{CE}/F_c^*)\right]^2 - \frac{F_{CE}/F_c^*}{c}}
\]

\[
F_{CE} = 1000 \cdot (1.6 \cdot 1.15) = 1840
\]

\[
F_c^* = 0.822 \cdot \frac{400000}{(99.5/3.5)^2} = 406.8
\]

\[
c = 0.8
\]

\[
C_p = 0.21
\]
Combined Stress in NDS example
Exterior stud wall under bending + axial compression

4. Calculate factored stresses

Compression Stress

Actual Stress
\[ f_c = \frac{P}{A} = \frac{900}{5.25} = 171.4 \text{ psi} \]

Factored Allowable Stress
\[ P^F = 1000(1.6)(1.15)(0.21) = 386.4 \text{ psi} \]

Combined Stress Calculation

\[
\left[ \frac{f_c}{f_c^A} \right]^2 + \frac{f_{bl}}{f_{bl}^A} \left( \frac{1}{1 - \left( \frac{f_c}{f_c^A} \right)} \right) \leq 1.0
\]

\[
\left[ \frac{171.4}{386.4} \right]^2 + \frac{876}{2139 \left( \frac{1}{1 - \left( \frac{171.4}{406.8} \right)} \right)}
\]

\[ 0.1967 + (0.4095)(1.728) = 0.1967 + 0.7077 = 0.9045 \leq 1.0 \text{ OK} \]