Steel Column Analysis and Design

- Failure Modes
- Effects of Slenderness
- Stress Analysis of Steel Columns
- Capacity Analysis of Steel Columns
- Design of Steel Columns

Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

\[ P_{cr} = \frac{\pi^2 AE}{(KL/r)^2} = \frac{\pi^2 IE}{KL^2} \]

- \( A \) = Cross sectional area (in\(^2\))
- \( E \) = Modulus of elasticity of the material (lb/in\(^2\))
- \( K \) = Stiffness (curvature mode) factor
- \( L \) = Column length between pinned ends (in.)
- \( r \) = radius of gyration (in.)

\[ r = \sqrt{\frac{I}{A}} \]

\[ I = Ar^2 \]

\[ f_{cr} = \frac{\pi^2 E}{(KL/r)^2} \leq F_{cr} \]
Analysis of Steel Columns

Conditions of an Ideal Column

- initially straight
- axially loaded
- uniform stress (no residual stress)
- uniform material (no holes)
- no transverse load
- pinned (or defined) end conditions

Short columns
Fail by material crushing
Plastic behavior

Intermediate columns
Crush partially and then buckle
Inelastic behavior
Local buckling – flange or web
Flexural torsional buckling - twisting

Long columns
Fail in Euler buckling
Elastic behavior

\[ \text{slenderness} = \frac{KL}{r} \]
Analysis of Steel Columns

Estimate of effective length factor, $K$

Estimate of $K$:

<table>
<thead>
<tr>
<th>TABLE C-A-7.1</th>
<th>Approximate Values of Effective Length Factor, $K$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td><strong>Buckled shape of column is shown by dashed line</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Theoretical $K$ value</strong></td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Recommended design value when ideal conditions are approximated</strong></td>
<td>0.66</td>
</tr>
<tr>
<td><strong>End condition code</strong></td>
<td>Rotation fixed and translation fixed</td>
</tr>
</tbody>
</table>

University of Michigan, TCAUP

Slide 5 of 23
Determining K factors by Alignment Charts

**Sidesway Inhibited:**
Braced frame
1.0 > K > 0.5

**Sidesway Uninhibited:**
Un-braced frame
unstable > K > 1.0

**More Pinned:**
If $I_c/L_c$ is large
and $I_g/L_g$ is small
The connection is more pinned

**More Fixed:**
If $I_c/L_c$ is small
and $I_g/L_g$ is large
The connection is more fixed

\[
G = \frac{\sum (EI/L)_{column}}{\sum (EI/L)_{beam}}
\]
Analysis of Steel Columns - LRFD

Euler equation:

\[ F_e = \frac{\pi^2 E}{(KL/r)^2} \]

Short & Intermediate Columns:

\[ F_{cr} = \left[ 0.658 \frac{F_y}{F_e} \right] F_y \]

Long Columns:

\[ F_{cr} = 0.877 F_e \]

Analysis of Steel Columns - LRFD

Data:
- Column – size, length
- Support conditions
- Material properties – Fy
- Factored load – Pu

Required:
- Pu ≥ φPn (pass)

1. Calculate slenderness ratios. Lc/r , Lc=KL
   The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate \( 4.71 \sqrt{E/F_y} \) and determine column type (short or long)
4. Calculate \( F_{cr} \) based on slenderness
5. Determine φPn and compare to Pu
6. If Pu ≥ φPn , then OK
   \[ F_{cr} = \left[ 0.658 \frac{F_y}{F_e} \right] F_y \] Short
   \[ F_{cr} = 0.877 F_e \] Long
Analysis of Steel Columns
pass / fail by ASD

Data:
• Column – size, length
• Support conditions
• Material properties – Fy
• Factored Load – Pu

Required:
• \( P_u \leq \phi P_n \) (pass)

2. Check slenderness ratio against upper limit of 200 (recommended)

3. Calculate transition slenderness
4.71 \( \frac{E}{F_y} \) and determine column type (short or long)

4. Calculate \( F_{cr} \) based on slenderness

5. Determine \( \phi P_n \) and compare to \( P_u \)

6. If \( P_u \leq \phi P_n \), then OK
Analysis of Steel Columns

**Capacity** by LRFD

Data:
- Column – size, length
- Support conditions
- Material properties – $F_y$

Required:
- Max load capacity

1. Calculate slenderness ratios.
   The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness $4.71 \sqrt{E/F_y}$ and determine column type (short or long)
4. Calculate $F_{cr}$ based on slenderness
5. Determine $\phi P_n$ and Compute allowable capacity: $P_u = \phi P_n$

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**Capacity Example 1**

Free standing column
Third floor studio space
Supports roof load = 20 psf DL + SL
snow $\approx$ 15lbs / FT depth

- W8 x 35' A-36
  - $A = 10.3 \text{ in}^2$
  - $r_x = 3.51 \text{ in}$
  - $r_y = 2.03 \text{ in}$
Capacity Example 1


2. Check slenderness ratio against upper limit of 200 (recommended)

3. Calculate transition slenderness $4.71 \sqrt{E/F_y}$ and determine column type (short or long)

4. Calculate $F_{cr}$ based on slenderness

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5. Determine $\phi P_n$ and Compute allowable capacity: $P_u = \phi P_n$

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Column nominal strength

$P_n = F_{cr} \phi y = 25.74 \text{ kN}$

$\phi P_u = 0.9 (25.74) = 23.17 \text{ kN} = P_u$

Load capacity

$P_u = 1.2 \phi (40 \times 16) = 238.6 \text{ kN}$

$P_u = 1.2 (32) + 1.6 (54) = 238.6 \text{ kN}$

$SL = 1251 \text{ kN}$

$F_{cr} \phi y = 40 \times 16 = 640 \text{ kN}$

$SL = 125100 + 1600 \text{ kN} = 78.2 \text{ kN}$
Quiz 5  Due Feb 20

If snow weighs about 15 LBS/FT depth, how deep could the snow get on the roof before you might want to leave?

snow ≈ 15 lbs / FT depth

Load capacity

\[ P_0 = 1.2 (32) + 1.4 \times 25 = 238.4 \text{ k} \]

\[ SL = 125 \times 1 \text{ k} \]

For \( k_r = \frac{40 \times 40}{1600} = 100 \text{ SF} \)

\[ SL = 125 \times 100 \times 1600 = 78.2 \text{ k SF} \]

Capacity Example 2 (long column)

<table>
<thead>
<tr>
<th>Buckling mode</th>
<th>( K_e )</th>
<th>( K_x )</th>
<th>( K_y )</th>
<th>( K_{xy} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W \times 35 )</td>
<td>0.65</td>
<td>0.80</td>
<td>1.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Slenderness \( y-y \)

\[ \frac{K_y d^3}{E} = \frac{0.8 \times (25)^3}{20.03} = 118.2 \]

\[ 4.71 \frac{L}{d} = 113 < 118.2 \Rightarrow \text{LOK} \]

Euler Buckling

\[ F_e = \frac{K_{xy} E (\frac{L}{d})^2}{(K_x)^2} = \frac{72 \times 20000}{118.2^2} = 20.47 \text{ kpsi} \]

Long Column Equation

\[ F_e = 0.877 \times 20.47 = 17.95 \text{ kpsi} \]

Column strength

\[ \phi_{p_n} = \phi \times F_e \times A_g = 0.9 \times (17,95)(0.3) = 54.4 \text{ k} \]
Design of Steel Columns with AISC Strength Tables

Data:
- Column – length
- Support conditions
- Material properties – $F_y$
- Applied load - $P_{actual}$

Required:
- Column Size

1. Enter table with height.
2. Read allowable load for each section to find the smallest adequate size.
3. Tables assume weak axis buckling. If the strong axis controls the length must be divided by the ratio $r_x/r_y$.
4. Values stop in table (black line) at slenderness limit, $KL/r = 200$
### AISC Critical Stress Table

**for previous example $Kl/r_y = 118.2$**

#### Table 4-22
Available Critical Stress for Compression Members

<table>
<thead>
<tr>
<th>$f_y$ = 36 ksi</th>
<th>39 ksi</th>
<th>42 ksi</th>
<th>46 ksi</th>
<th>50 ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_p$ = 20 ksi</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>205.0</td>
<td>23.2</td>
<td>29.6</td>
<td>34.4</td>
<td>38.8</td>
</tr>
<tr>
<td>206.0</td>
<td>23.2</td>
<td>29.6</td>
<td>34.4</td>
<td>38.8</td>
</tr>
<tr>
<td>207.0</td>
<td>23.2</td>
<td>29.6</td>
<td>34.4</td>
<td>38.8</td>
</tr>
<tr>
<td>208.0</td>
<td>23.2</td>
<td>29.6</td>
<td>34.4</td>
<td>38.8</td>
</tr>
<tr>
<td>209.0</td>
<td>23.2</td>
<td>29.6</td>
<td>34.4</td>
<td>38.8</td>
</tr>
<tr>
<td>210.0</td>
<td>23.2</td>
<td>29.6</td>
<td>34.4</td>
<td>38.8</td>
</tr>
<tr>
<td>211.0</td>
<td>23.2</td>
<td>29.6</td>
<td>34.4</td>
<td>38.8</td>
</tr>
<tr>
<td>212.0</td>
<td>23.2</td>
<td>29.6</td>
<td>34.4</td>
<td>38.8</td>
</tr>
<tr>
<td>213.0</td>
<td>23.2</td>
<td>29.6</td>
<td>34.4</td>
<td>38.8</td>
</tr>
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<td>214.0</td>
<td>23.2</td>
<td>29.6</td>
<td>34.4</td>
<td>38.8</td>
</tr>
<tr>
<td>215.0</td>
<td>23.2</td>
<td>29.6</td>
<td>34.4</td>
<td>38.8</td>
</tr>
</tbody>
</table>

$Q_{cr} = 1.87 \times (f_y/r_y) = 90$
Steel Frame Construction

University of Michigan – North Quad